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Sprays: models, interactions and collisions.

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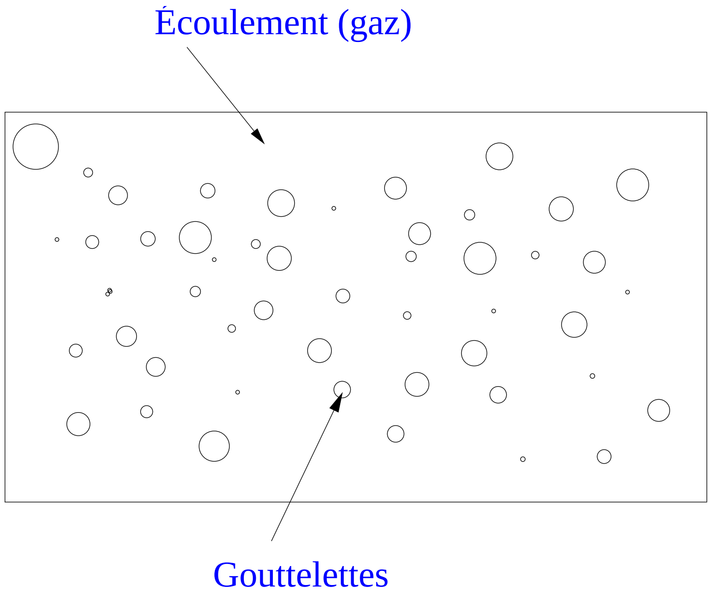

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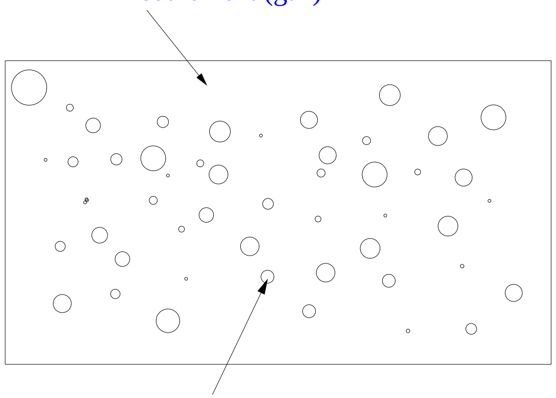
I Gas-particles models and interactions

Sprays = gas with droplets



- P. O'Rourke. *Collective drop effects on vaporizing liquid sprays*. Thèse, Princeton University, 1981.
- F.A. Williams. *Combustion theory*. Benjamin Cummings, 1985.
- G. Dufour. *Modélisation multi-fluide eulérienne pour les écoulements diphasiques à inclusions dispersées*. Thèse, Université Paul Sabatier Toulouse III, 2005.

Sprays : fluid part

- Compressible fluid flows

  Macroscopic quantities (depending on (t, x))
  - ⋆ density $\rho_g(t, x)$
  - ⋆ velocity $u_g(t, x)$
  - ⋆ total energy $E_g(t, x)$
  - ⋆ pressure $p(t, x)$

  Euler equations

$$
\begin{aligned}
\partial_t \rho_g + \nabla_x \cdot (\rho_g u_g) &= 0, \\
\partial_t (\rho_g u_g) + \nabla_x \cdot (\rho_g u_g \otimes u_g) + \nabla_x p &= 0, \\
\partial_t (\rho_g E_g) + \nabla_x \cdot ((\rho_g E_g + p) u_g) &= 0, \\
p_g = P(\rho_g, e_g) & \quad , \\
E_g = e_g + \frac{1}{2} {u_g}^2 & \quad .
\end{aligned}
$$

Sprays : particles

- Particles : described through Boltzmann theory

  Probability density function $f(t, x, u_p, e_p ...)$: x position, u_p velocity, e_p internal energy...

  Vlasov-Boltzmann equation

$$\partial_t f + \underbrace{\nabla_x \cdot (f u_p)}_{\text{transport (advection)}} + \underbrace{\nabla_{u_p} \cdot (f \Gamma) + \nabla_{e_p} \cdot (f \phi)}_{\text{acceleration , thermic exchanges}} = \underbrace{Q(f, f)}_{\text{collisional term}}$$

Sprays : equations

- Spray: coupling of a fluid and particles
- One more unknown: α, gas volume fraction ($\alpha(t,x)$)
- Sprays classification

$$
\begin{aligned}
\partial_t(\alpha\,\rho_g) + \nabla_x\cdot(\alpha\,\rho_g\,u_g) &= 0,\\
\partial_t(\alpha\,\rho_g\,u_g) + \nabla_x\cdot(\alpha\,\rho_g\,u_g\otimes u_g) + \nabla_x p &= \iint_{u_p,e_p} -m_p\,\Gamma\,f\,du_p de_p,\\
\partial_t(\alpha\rho_g E_g) + \nabla_x\cdot\left(\alpha\rho_g\left(E_g + \frac{p}{\rho_g}\right)\right) + p\partial_t\alpha &= \iint_{u_p,e_p} -m_p\,f(\Gamma + \frac{\nabla_x p}{\rho_p})\cdot u_p du_p de_p\\
&- \iint_{u_p,e_p} m_p f\phi, du_p de_p,\\
\partial_t f + u_p\cdot\nabla_x f + \nabla_{u_p}\cdot(f\,\Gamma) + \nabla_{e_p}\cdot(f\,\phi) &= Q(f,f),
\end{aligned}
$$

very thin	thin	thick
$\alpha = 1$	$\alpha = 1$	$\alpha = 1 - \int_{v,e_p} \frac{4}{3}\pi r_p^3\, f\, dv de_p$
$m_p\,\Gamma = D_p(u_g - u_p)$	$m_p\,\Gamma = D_p(u_g - u_p)$	$m_p\,\Gamma = D_p\,(u_g - u_p) - \frac{4}{3}\pi r_p^3\,\nabla_x p$
$Q(f,f) = 0$		

Models of coupling

- Momentum

  Γ: acceleration of particles

$$m_p\Gamma = \text{pressure + drag force}$$
$$= -\frac{4}{3}\pi r_p^3 \nabla_x p - D_p(u_p - u_g)$$

- Thermic exchanges

  ϕ: thermic exchanges by mass unit

$$m_p\phi = 4\pi r_p \lambda Nu(T_g - T_p)$$

m_p , r_p: mass, radius of particles
D_p , Nu , λ: drag force coefficient , Nusselt number, thermal conductivity

More physics on particles

- Break-up depending on the Weber number: $We = \dfrac{2r\rho_g|u_g - u_p|^2}{\sigma_p}$

  1. pour $We_c \leq We_* \leq 18$ on a:
  $$\frac{r_{32}(u_{p_*}, r_*, e_{p_*})}{r_*} = 0.32(We_*)^{-\frac{1}{3}} \left(\frac{4.1}{(We_* - 12)^{1/4}}\right)^{2/3},$$
  2. pour $18 \leq We_* \leq 45$ on a:
  $$\frac{r_{32}(u_{p_*}, r_*, e_{p_*})}{r_*} = 0.32(We_*)^{-\frac{1}{3}} \left(\frac{2.45\sqrt{We_* - 12} - 1.9}{(We_* - 12)^{1/4}}\right)^{2/3},$$
  3. pour $45 \leq We_*$ on a: $\dfrac{r_{32}(u_{p_*}, r_*, e_{p_*})}{r_*} = 0.32(We_*)^{-\frac{1}{3}} \left(\dfrac{12.2}{(We_* - 12)^{1/4}}\right)^{2/3}.$

- Compressibility of the droplets

$$m_p\Gamma = -\frac{m_p}{\rho_p}\nabla_x p - D_p(u_p - u_g)$$

$$m_p\phi = 4\pi r\lambda Nu(T_g - T_p) + \frac{p m_p}{\rho_p^2}\frac{d\rho_p}{dt} \quad \left(\frac{d\rho_p}{dt} = \partial_t \rho_p + u_p \cdot \nabla_x \rho_p + \Phi \partial_{e_p} \rho_p\right)$$

- Abrasion, combustion, chemical reactions ...

Gas-particles system

- Gas equations

$$\partial_t(\alpha\rho_g) + \nabla_x \cdot (\alpha\rho_g u_g) = 0$$

$$\partial_t(\alpha\rho_g u_g) + \nabla_x \cdot (\alpha\rho_g u_g \otimes u_g) + \nabla_x p = -\int_{u_p,e_p} m_p \Gamma f du_p de_p$$

$$\partial_t(\alpha\rho_g E_g) + \nabla_x \cdot \left(\alpha\rho_g \left(E_g + \frac{p}{\rho_g}\right) u_g\right) + p\partial_t\alpha =$$

$$\int_{u_p,e_p} -(m_p\Gamma + \frac{m_p}{\rho_p}\nabla_x p) \cdot u_p f du_p de_p - \int_{u_p,e_p} 4\pi r\lambda Nu(T_g - T_p) f du_p de_p$$

- Vlasov-Boltzmann equation

$$\partial_t f + u_p \cdot \nabla_x f + \nabla_{u_p} \cdot (f\Gamma) + \partial_{e_p}(f\phi) = Q(f,f)$$

- Equations of State

$$T_g = T_1(\rho_g, e_g) \quad , \quad p = P_1(\rho_g, e_g) = P_2(\rho_p, e_p) \quad , \quad T_p = T_2(\rho_p, e_p)$$

Existing results

Spray:

- K. Domelevo and J-M. Roquejoffre. Existence and stability of travelling wave solutions in a kinetic model of two-phase flows. *Comm. Partial Differential Equations*, 24(1-2):61–108, 1999.

  Coupling between Burgers equation and Vlasov equation.

- C. Baranger and L. Desvillettes. Coupling Euler and Vlasov equations in the context of sprays : local smooth solutions. *J. Hyperbolic Differ. Equ.*, 3:1–26, 2006.

  Coupling between isentropic Euler equations and Vlasov equation.

- J.Mathiaud. Local smooth solutions of a thin spray model with collisions. *Mathematical Models and Methods in Applied Sciences"*, To appear, 2010.

  Coupling between Euler equations and Vlasov-Boltzmann equation.

II Collisions and other particles phenomena

$$particle^{(1)} + particle^{(2)} \longrightarrow particle^{(3)} + particle^{(4)}$$

Particle mass: m_p
Particle velocity: u_p
Internal energy: e_p

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Internal energy: e_p

- $u_{p_3} = \frac{u_{p_1} + u_{p_2}}{2} + \beta \frac{|u_{p_1} - u_{p_2}|}{2} \sigma$: post-collisionnal velocity
- $u_{p_4} = \frac{u_{p_1} + u_{p_2}}{2} - \beta \frac{|u_{p_1} - u_{p_2}|}{2} \sigma$: post-collisionnal velocity
- $\Delta E'_c = \frac{1}{2}({u_{p_1}}^2 + {u_{p_2}}^2 - {u_{p_3}}^2 - {u_{p_4}}^2) = \frac{1-\beta^2}{4}(u_{p_1} - u_{p_2})^2 > 0$: loss of kinetic energy

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- $e_{p_3} = \dfrac{2-a}{2} e_{p_1} + \dfrac{a}{2} e_{p_2} + b\Delta E'_c$: post-collisional internal energy
- $e_{p_4} = \dfrac{2-a}{2} e_{p_2} + \dfrac{a}{2} e_{p_1} + (1-b)\Delta E'_c$: post-collisional internal energy

a: parameter of internal energy exchanges ($0 \leq a \leq 1$)
b: parameter of transfer of kinetic energy into internal energy ($0 \leq b \leq 1$)

Estimates on a , b and β

Characteristic time of collision: $\Delta T_{coll} = \dfrac{2r_p}{|u_{p_1} - u_{p_2}|}$

Characteristic time of thermal exchanges: $\Delta T_{th} = \dfrac{Cm_p}{4\pi r\lambda}$

Characteristic time of viscous effects: $\tau_c = 1 \Big/ \frac{10\mu_p}{\rho_p r^2}$

$b = 1/2$ for particles of same masses

After linearization of the problem:

$$a = 1 - \exp\left(- \frac{8\pi\lambda_p {r_p}^2}{Cm_p |u_{p_1} - u_{p_2}|} \right)$$

Typically, $a \sim 0.71$.

$$\beta = \exp\left(- \frac{\Delta T_{coll}}{\tau_c} \right).$$

Typically, $\beta \sim 0.89$.

particles number: 2.10^{10}	density: $7310\, kg.m^{-3}$
radius: $10^{-6}\, m$	velocity: $200\, m.s^{-1}$
internal energy: $10^6\, J.kg^{-1}$	temperature: $300\,^\circ C$
specific heat: $200\, J.kg^{-1}.K^{-1}$	dynamic viscosity: $8.10^{-3}\, Pa.s$
surface tension: $0.56\, N.m^{-1}$	thermal conductivity: $60\, W.m^{-1}.K^{-1}$

III Hydrodynamic limit of the system

- Hilbert expansion:
  - ⋆ S. Chapman and T.G. Cowling. *The mathematical theory of non uniform gases*. Cambridge Mathematical Library, 1990.
  - ⋆ F. Golse and L. Saint-Raymond. The Navier-Stokes limit of the Boltzmann equation for bounded collision kernels. *Invent. Math.*, 155(1):81–161, 2004.

- Granular flows:
  - ⋆ P.K. Haff. Grain flow as a fluid mechanical phenomenon. *J. Fluid. Mech.*, 134, 1983.
  - ⋆ S. Mischler and C. Mouhot. Cooling process for inelastic Boltzmann equations for hard spheres, part II: Self-similar solutions and tail behavior. *J. Statis. Phys.*, To appear, 2006.

Fluid of particles:

- $\rho = \frac{1}{1-\alpha} \int_{u_p,e_p} f m_p du_p de_p$: density
- $v = \frac{1}{(1-\alpha)\rho} \int_{u_p,e_p} f m_p u_p du_p de_p$: velocity
- $e_c = \frac{1}{(1-\alpha)\rho} \int_{u_p,e_p} \frac{1}{2} f m_p u_p^2 du_p de_p$: mesoscopic kinetic energy
- $e = \frac{1}{(1-\alpha)\rho} \int_{u_p,e_p} f m_p e_p du_p de_p$: mesoscopic internal energy
- $E_p = e + e_c$: total energy
- $p' = \frac{1}{1-\alpha} \int_{u_p,e_p} f m_p (v - u_p) \otimes (v - u_p) du_p de_p$: Reynolds stress
- $q = \frac{1}{1-\alpha} \int_{u_p,e_p} f m_p (v - u_p)^2 (u_p - v) du_p de_p$: Thermal fluctuation

Two-phase flows system (unclosed)

Mass conservation:

$$\partial_t(\alpha\rho_g) + \nabla_x \cdot (\alpha\rho_g u_g) = 0$$

$$\partial_t((1-\alpha)\rho) + \nabla_x \cdot ((1-\alpha)\rho v) = 0$$

$\rho = \rho_p = cte$ (incompressible droplets)

Momentum conservation:

$$\partial_t(\alpha\rho_g u_g) + \nabla_x \cdot (\alpha\rho_g u_g \otimes u_g) + \alpha\nabla_x p = - \int_{u_p,e_p} D_p(u_g - u_p) f du_p de_p$$

$$\partial_t((1-\alpha)\rho v) + \nabla_x \cdot ((1-\alpha)\rho v \otimes v) + (1-\alpha)\nabla_x p + \nabla_x \cdot ((1-\alpha)p') = - \int_{u_p,e_p} D_p(u_p - u_g) f du_p de_p$$

Total energy:

$$\partial_t(\alpha\rho_g E_g) + \nabla_x \cdot \left(\alpha\rho_g \left(E_g + \frac{p}{\rho_g}\right) u_g\right) + p\partial_t\alpha =$$

$$\int\limits_{u_p,e_p} D_p(u_p - u_g)\cdot u_p f du_p de_p - \int\limits_{u_p,e_p} 4\pi r_p \lambda Nu(T_g - T_p) f du_p de_p$$

$$\partial_t\left((1-\alpha)\rho E_p\right) + \nabla_x \cdot \left((1-\alpha)\rho \left(E_p + \frac{p+p'}{\rho}\right) v\right) + p\partial_t(1-\alpha) + \nabla_x \cdot ((1-\alpha)q) =$$

$$-\int\limits_{u_p,e_p} D_p(u_p - u_g)\cdot u_p f du_p de_p + \int\limits_{u_p,e_p} 4\pi r_p \lambda Nu(T_g - T_p) f du_p de_p$$

Equations of state:

$$p(t,x) = P_1(\rho_g(t,x), e_g(t,x)) \ , \ T_g(t,x) = T_1(\rho_g(t,x), e_g(t,x)) \ , \ T_p = T_2(e_p) \ ,$$

p' and q unknowns.

Non-Dimensionnal analysis

$$\partial_{\tilde{t}}\tilde{f} + \tilde{u}_p \cdot \nabla_{\tilde{x}}\tilde{f} + \nabla_{\tilde{u}_p} \cdot (\tilde{f}\tilde{\Gamma}) + \partial_{\tilde{e}_p}(\tilde{f}\tilde{\phi}) = \frac{1}{\varepsilon}Q(\tilde{f},\tilde{f})$$

with:

- T (resp. L): characteristic time (resp. length) of the gas
- N: typical number of particles in L^3
- σ: mean free path $N = \dfrac{L^3}{{r_p}^2\sigma}$
- $\varepsilon = \dfrac{\sigma}{L}$: Knudsen number
- Typical Nusselt numbers are equal to 0.083

Inelastic Hydrodynamic Limit :

- Homogeneous case: $\partial_t f = Q(f,f)$

$$T(t) = \iint f(t,u_p,e_p)\frac{1}{3}m_p(u_p - v)^2 du_p de_p / \iint f(t,u_p,e_p) m_p du_p de_p,$$

Haff's law: $T(t) \underset{+\infty}{\sim} \dfrac{1}{t^2}$.

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- Internal energy:

$$g(t) = \iint f(t,u_p,e_p) m_p \left|e_p - e(t)\right|^2 du_p de_p / \iint f(t,u_p,e_p) m_p du_p de_p.$$

$$g'(t) \quad \sim \quad (1-\alpha)\left(-a(1-\frac{a}{2})\sqrt{3}\frac{3}{r}T(t)^{1/2}g(t) + \frac{1}{4}\left(\frac{1-\beta^2}{4}\right)^2 \frac{3}{r}\left(3T(t)\right)^{5/2}\right),$$

$$g(t) \underset{+\infty}{\sim} \frac{1}{t^c} \text{ with } c = \min(4, \frac{3\sqrt{\pi}\sqrt{3}a(1-a/2)}{4(1-\beta^2)}).$$

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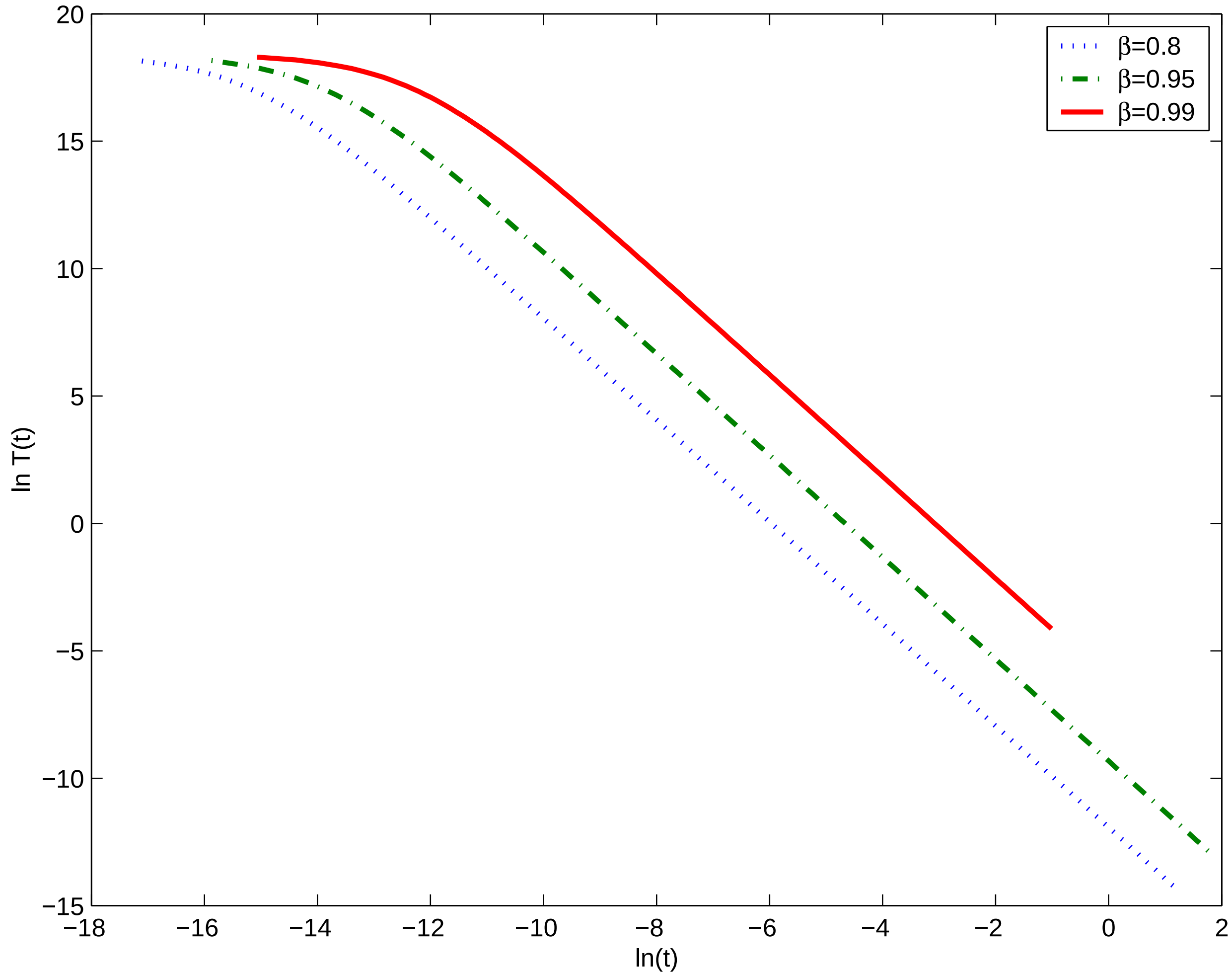
$$g(t) \underset{+\infty}{\sim} \frac{1}{t^c} \text{ with } c = \min(4, \frac{3\sqrt{\pi}\sqrt{3}a(1-a/2)}{4(1-\beta^2)}).$$

- Formally, we get:

$$f_\varepsilon(t,u_p,e_p) \underset{t\to+\infty}{\longrightarrow} Z\delta_{e_p=e}(e_p) \otimes \delta_{u_p=v}(u_p)$$

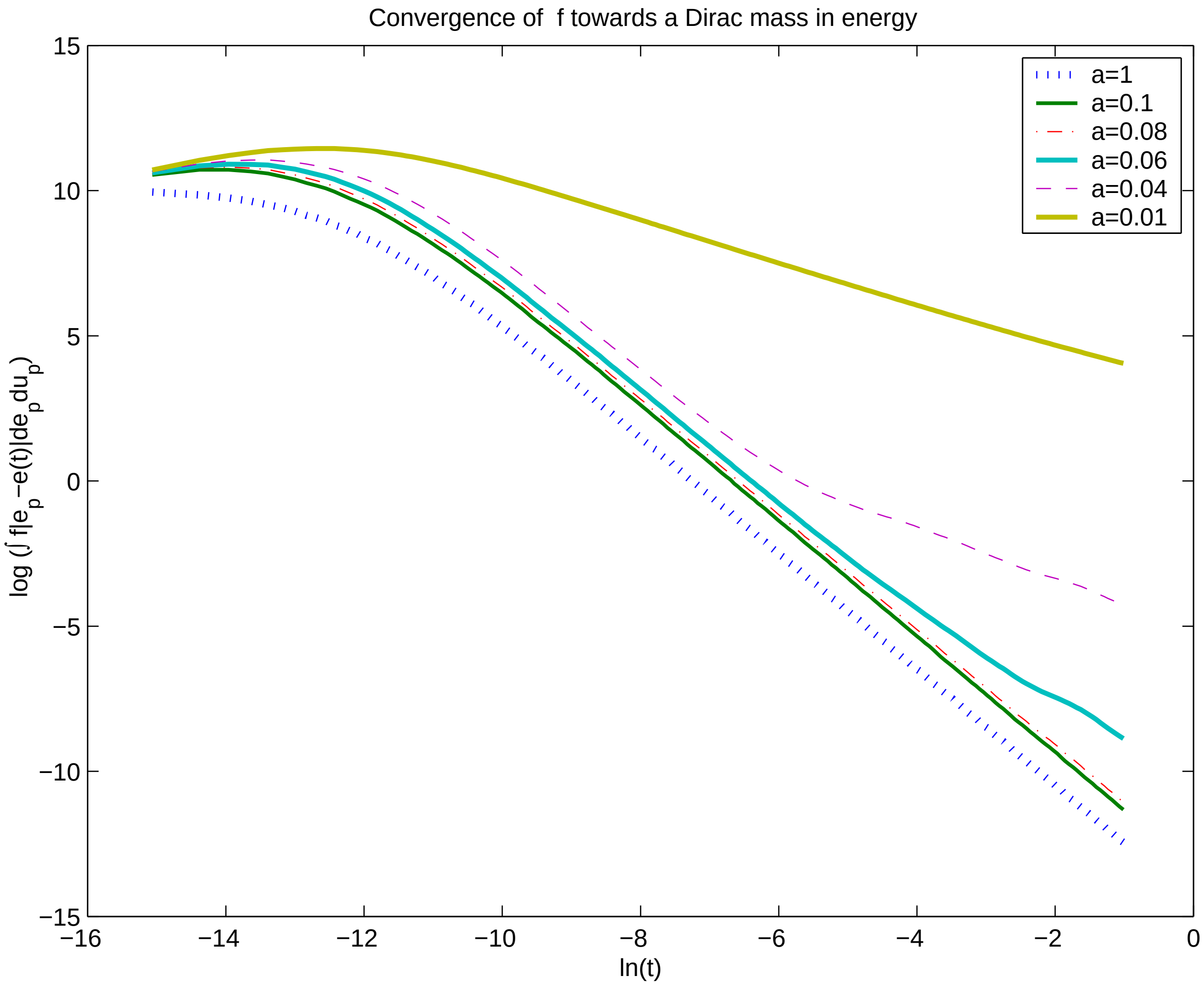
Homogeneous code $\partial_t f = Q(f,f)$



Convergence towards a Dirac mass in velocity for different β:

$\ln\left(\iint f(t,u_p,e_p)|u_p|^2 de_p du_p / \iint f(t,u_p,e_p) de_p du_p\right)$ as a function of $\ln t$

Homogeneous code $\partial_t f = Q(f,f)$



Convergence towards a Dirac mass in internal energy for different a:

$\ln\left(\iint f(t,u_p,e_p)|e_p - e(t)|de_p du_p / \iint f(t,u_p,e_p)de_p du_p\right)$ as a function of $\ln t$ for different a

Inelastic hydrodynamic limit

When particles are numerous enough, f is more or less equivalent to:

$$f_\varepsilon(t,x,u_p,e_p) \underset{\varepsilon\to 0}{\to} Z(t,x)\delta_{e_p=e}(e_p)\otimes\delta_{u_p=v}(u_p)$$

<u>Mass conservation:</u>

$$\partial_t(\alpha\rho_g)+\nabla_x\cdot(\alpha\rho_g u_g)=0$$
$$\partial_t((1-\alpha)\rho)+\nabla_x\cdot((1-\alpha)\rho v)=0$$
$$\rho=\rho_p=cte$$

<u>Momentum conservation:</u>

$$\partial_t(\alpha\rho_g u_g)+\nabla_x\cdot(\alpha\rho_g u_g\otimes u_g)+\alpha\nabla_x p=-\frac{1}{2m_p}(1-\alpha)\rho D_p\left(u_g-v\right)$$
$$\partial_t((1-\alpha)\rho v)+\nabla_x\cdot((1-\alpha)\rho v\otimes v)+(1-\alpha)\nabla_x p=\frac{1}{2m_p}(1-\alpha)\rho D_p\left(u_g-v\right)$$

Total energy equations:

$$\partial_t(\alpha\rho_g E_g) + \nabla_x \cdot \left(\alpha\rho_g \left(E_g + \frac{p}{\rho_g}\right) u_g\right) + p\partial_t\alpha =$$
$$- \frac{1}{2m_p}(1-\alpha)\rho D_p\,(u_g - v)\cdot v - 4\pi\, r_p\,\lambda\, Nu\,(T_g - T_p)\;\frac{(1-\alpha)\rho}{m_p}$$

$$\partial_t\left((1-\alpha)\rho E_p\right) + \nabla_x \cdot \left((1-\alpha)\rho\left(E_p + \frac{p}{\rho}\right) v\right) + p\partial_t(1-\alpha) =$$
$$\frac{1}{2m_p}(1-\alpha)\rho D_p\,(u_g - v)\cdot v + 4\pi\, r_p\,\lambda\, Nu\,(T_g - T_p)\;\frac{(1-\alpha)\rho}{m_p}$$

Equations of state:

$$p = P_1(\rho_g, e_g)\ , \qquad T_g = T_1(\rho_g, e_g)\ ,\ \ T_p = T_p(e)$$
$$(p' = 0, \qquad q = 0)$$

Numerical resolution

- Splitting method:
  - ⋆ gas: Lagrange + remap
  - ⋆ particles: characteristics + spurious collisions (Bird's method)

- Numerical data
  - ⋆ 100000 cells of length $4.10^{-5}m$
  - ⋆ collision time: $2.10^{-8}s$
  - ⋆ time of convergence in velocity: $9.10^{-7}s$
  - ⋆ time of convergence in internal energy: $1.10^{-6}s$
  - ⋆ characteristic time of the gas: $10^{-5}s$

Inelastic effects: elastic collisions

Inelastic effects: inelastic collisions

A diffusive model for computing collisions (1)

(This work is done with A. Champmartin and L. Desvillettes)

$$\partial_t f = Q(f, f), \tag{1}$$

We want a Fokker-Planck like equation (less costly to compute) which solves more or less the Boltzmann equation for hard spheres. We define the moments:

$$\frac{1-\alpha}{\frac{4}{3}\pi r^3} = \int f\,dv; \qquad \frac{1-\alpha}{\frac{4}{3}\pi r^3}\,u_f = \int f\,v\,dv; \qquad 3\frac{1-\alpha}{\frac{4}{3}\pi r^3}\,T_f = \int f\,|v-u_f|^2\,dv.$$

We now suppose that $u_f = 0$ (Galilean invariance). In weak formulation, we get for Boltzmann equation

$$\partial_t \int f\,\phi\,dv = \int_{v\in\mathbb{R}^3}\int_{v_*\in\mathbb{R}^3}\int_{\sigma\in S^2} f(v)\,f(v_*)\,r^2\,|v-v_*| \times \Big\{\phi(v') - \phi(v)\Big\}\,d\sigma\,dv_*\,dv.$$

We want mass, momentum and energy conservation $\phi = 1, v, |v|^2$. We also want to know the behavior of $\phi(v) = v_i\,v_j$ (i.-e. a directional temperature).

A diffusive model for computing collisions (2)

$$\partial_t \int f\, v_i\, v_j\, dv = 4\pi r^2 \sqrt{6\, T_f} \int_{v\in\mathbb{R}^3} \int_{v_*\in\mathbb{R}^3} f(v)\, f(v_*)$$

$$\times \left\{ \int_{\sigma\in S^2} \left(\frac{v_i + v_{i*}}{2} + \frac{|v - v_*|}{2}\, \sigma_i\right) \left(\frac{v_j + v_{j*}}{2} + \frac{|v - v_*|}{2}\, \sigma_j\right) \frac{d\sigma}{4\pi} - v_i v_j \right\} dv_*\, dv$$

$$= 4\pi r^2 \sqrt{6\, T_f} \int_{v\in\mathbb{R}^3} \int_{v_*\in\mathbb{R}^3} f(v)\, f(v_*)$$

$$\times \left\{ \frac{1}{4} \left(v_i v_j + v_i v_{j*} + v_{i*} v_j + v_{i*} v_{j*}\right) - v_i v_j + \frac{1}{4} \left(|v|^2 + |v_*|^2 - 2v\cdot v_*\right) \int \sigma_i \sigma_j \frac{d\sigma}{4\pi} \right\} dv_*\, dv$$

$$= 4\pi r^2 \sqrt{6\, T_f}\, \frac{1-\alpha}{\frac{4}{3}\pi r^3} \left\{ -\frac{1}{2} \int f\, v_i\, v_j\, dv + \frac{1}{6}\, \delta_{ij} \left(3 \frac{1-\alpha}{\frac{4}{3}\pi r^3}\, T_f\right) \right\}.$$

using $|v - v_*| \sim \sqrt{\iint_{v,v_*} f f^* |v - v_*|^2 dv dv_*} = \sqrt{6\, T_f}$

Fokker-Planck equation

We now introduce a Fokker-Planck equation

$$\partial_t f = \nu_f \nabla_v \cdot (T_f \nabla_v f + (v - u_f) f), \qquad (2)$$

in which mass, momentum and energy are conserved.
There remains to compute ν_f. Using equation (2), we get

$$\partial_t \int f\, v_i\, v_j\, dv = \nu_f \sum_k \int_{v \in \mathbb{R}^3} -\partial_{v_k}(v_i v_j)\, [T_f\, \partial_{v_k} f + v_k\, f]\, dv$$

$$= -\nu_f \sum_k \int_{v \in \mathbb{R}^3} \Big[- T_f\, (\delta_{ik}\delta_{jk} + \delta_{jk}\delta_{ik})\, f + (\delta_{ik}\, v_j v_k + \delta_{jk}\, v_i v_k)\, f \Big]\, dv$$

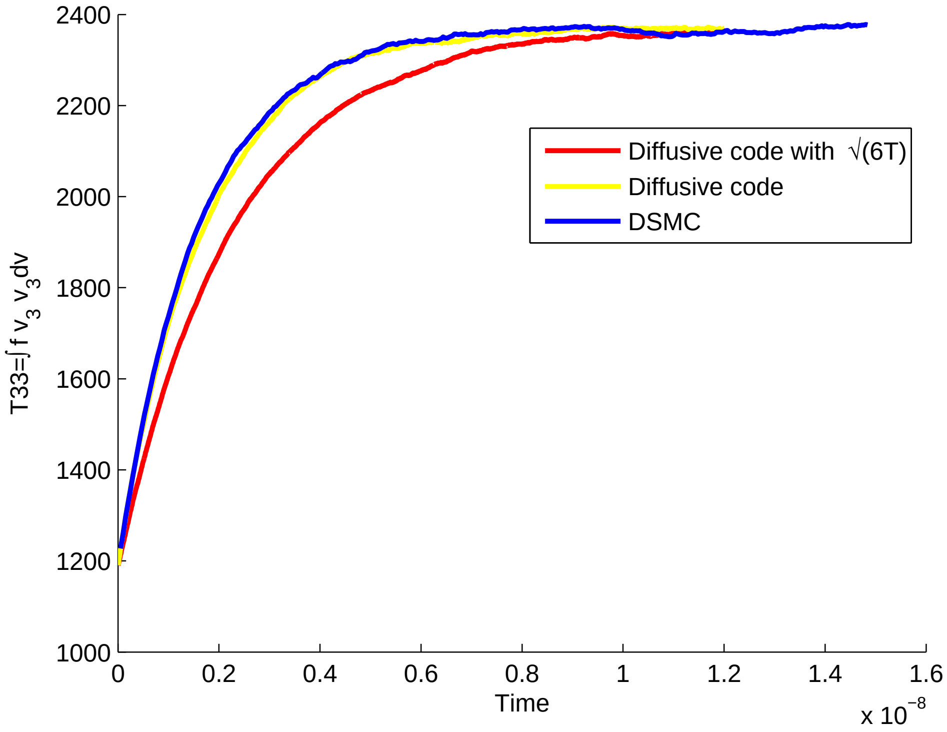
$$= -\nu_f\, (-2\delta_{ij}\, T_f\, \frac{1-\alpha}{\frac{4}{3}\pi r^3} + 2 \int_{v \in \mathbb{R}^3} v_i v_j\, f\, dv).$$

Identifying one gets:

$$\nu_f = \frac{3}{4r}(1-\alpha)\sqrt{6\, T_f}.$$

with r mean radius of the particles.

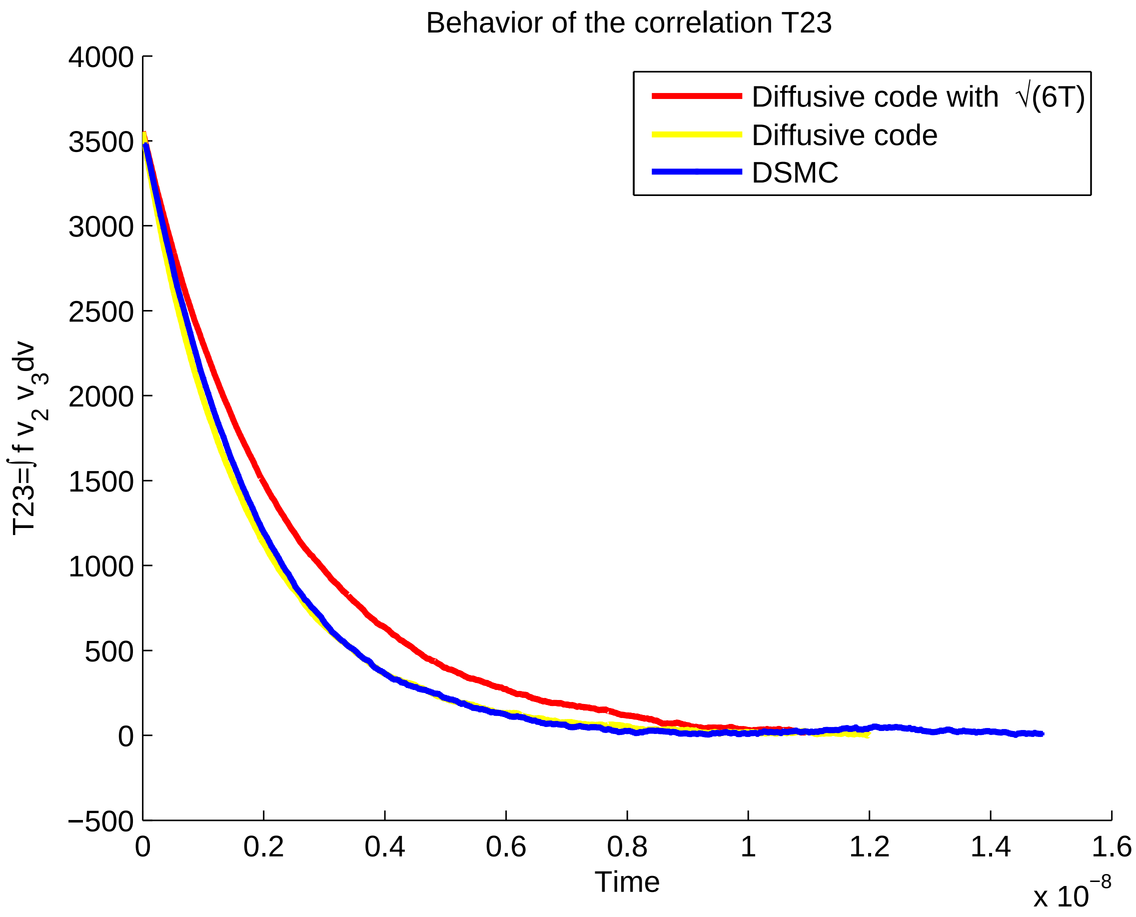
DSMC vs FK: $\partial_t f = Q(f,f)$



The other version of the diffusive code is due to another closure of $|v - v_*|$.

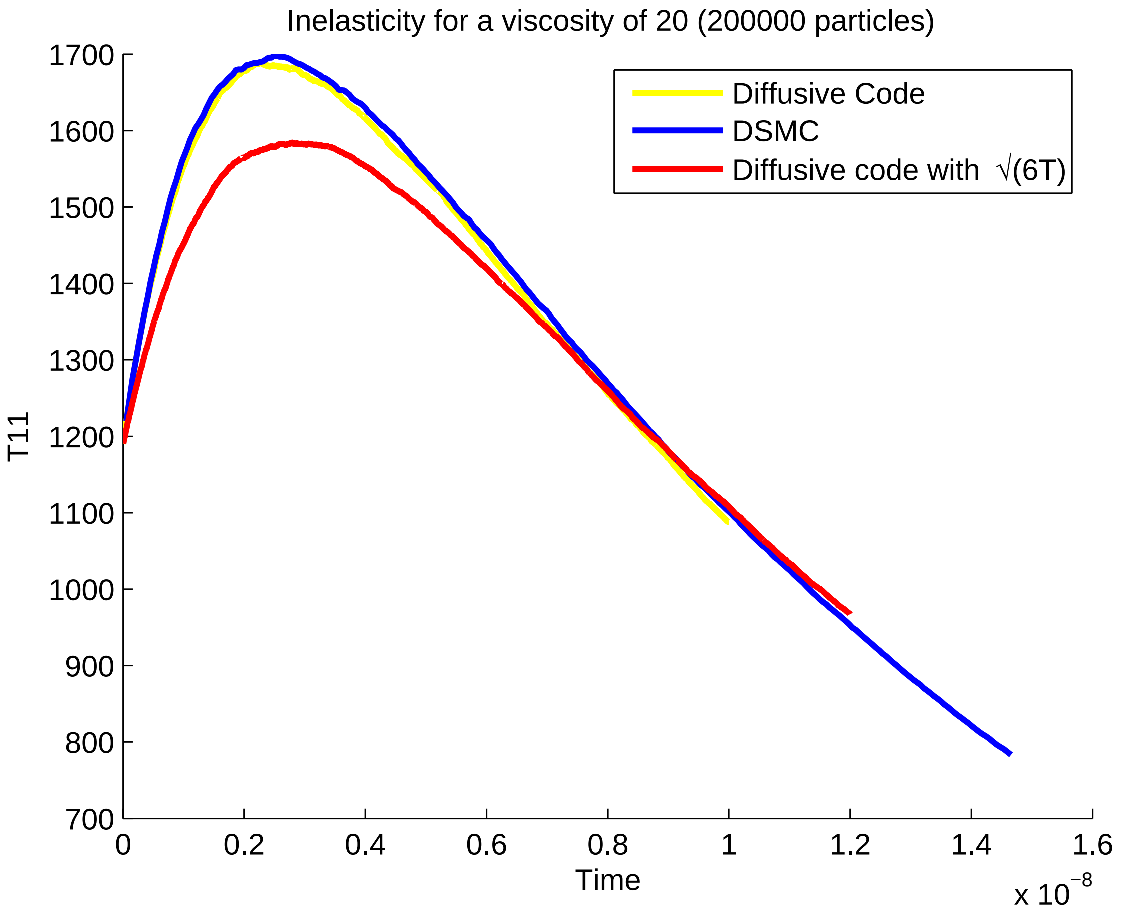
$$\sqrt{\frac{(3(T_{11}^2 + T_{22}^2 + T33^2) + 2(T_{11}T_{22} + T_{33}T_{11} + T_{22}T_{33}))}{9T^2}}\sqrt{6T}$$

DSMC vs FK: $\partial_t f = Q(f, f)$

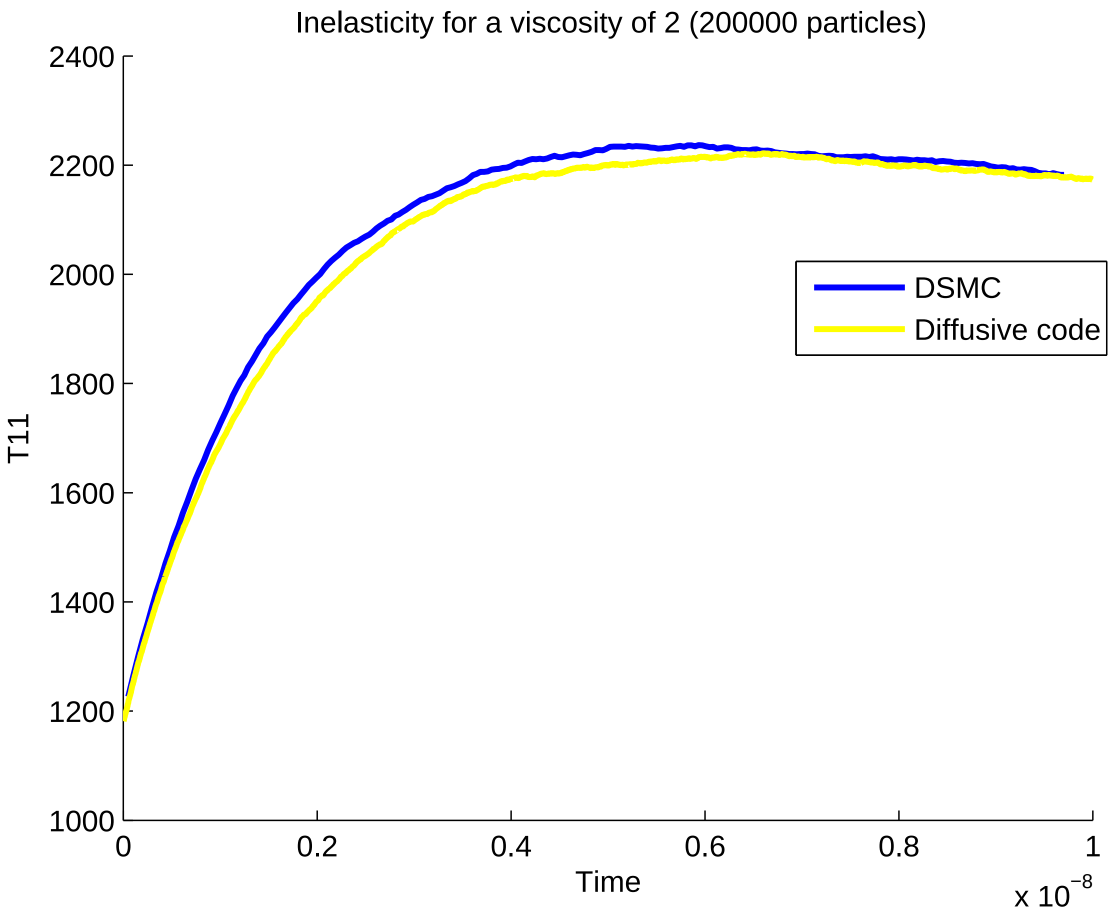


DSMC vs FK inélastique: $\partial_t f = Q(f, f)$

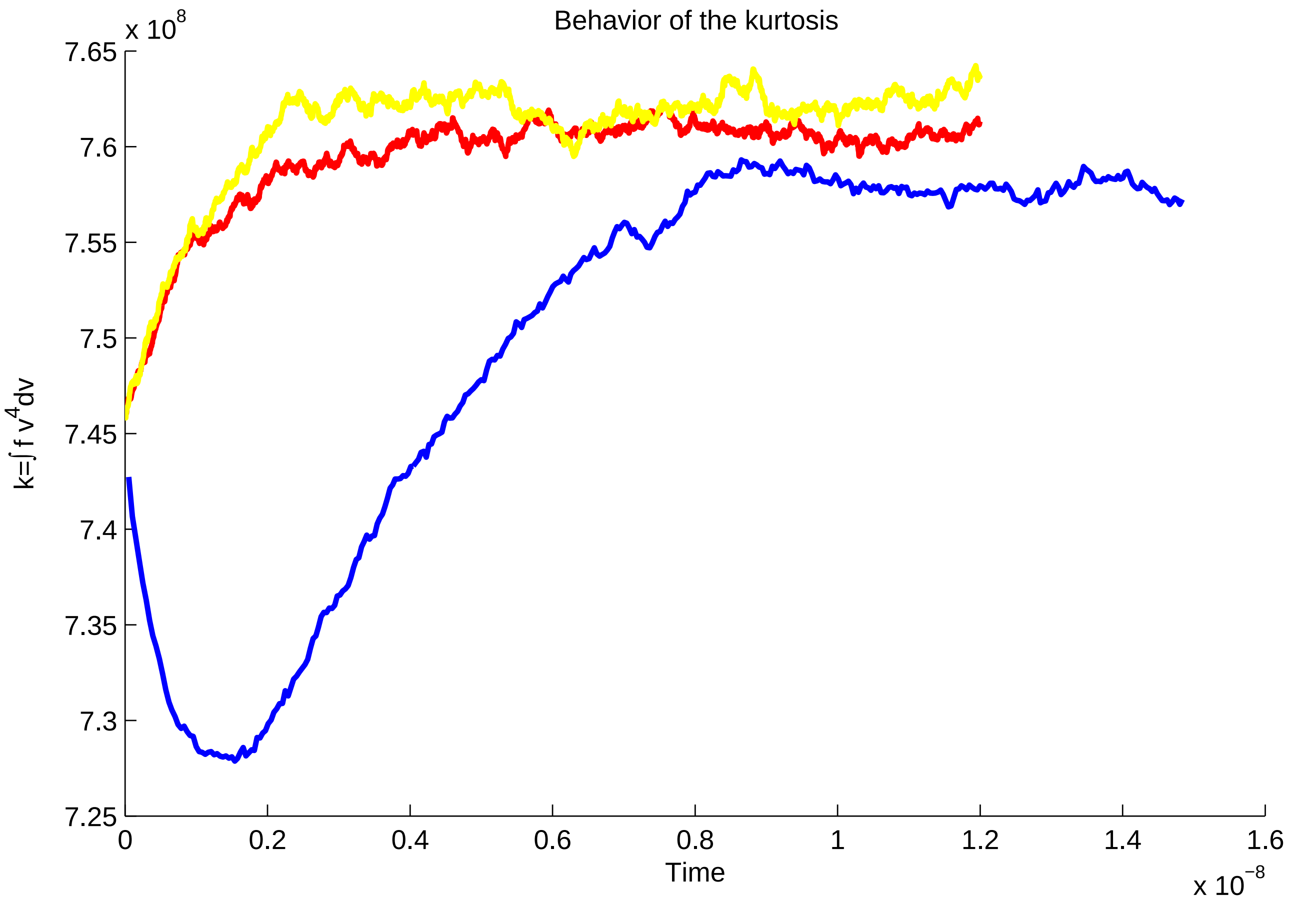
$$\partial_t f = \nu_f \nabla_v \cdot (\beta T_f \nabla_v f + (v - u_f) f), \tag{3}$$



DSMC vs FK inélastique: $\partial_t f = Q(f,f)$



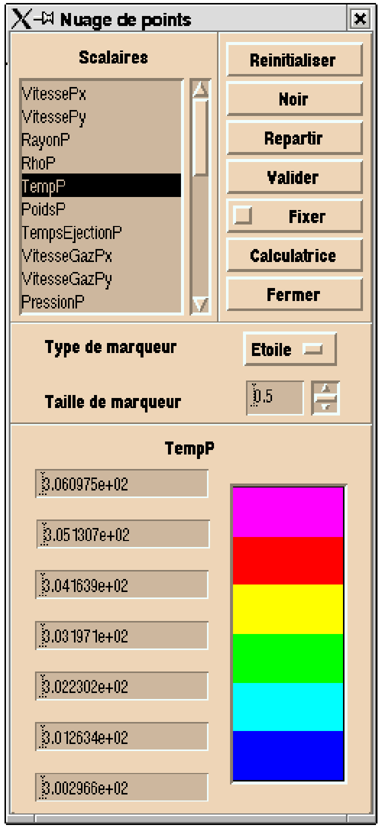
DSMC vs FK inélastique: $\partial_t f = Q(f,f)$

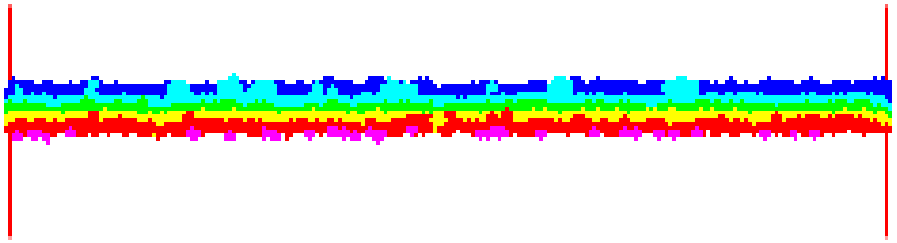


Conclusion & perspectives

- Study of inelasticity and equilibrium
- More physics in the code
- Dealing better with the volume fraction

Without thermic exchanges





With thermic exchanges