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A new model for spray interactions.

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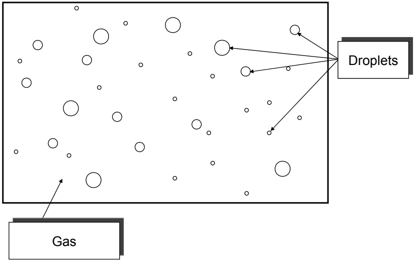
1. Context
   - Diesel engines,
   - Ariane boosters,
   - Aerosols.

2. Goal :
   - Having a test code able to deal with various applications,
   - Treating cases with numerous droplets of different sizes,
   - Establishing test cases.

The basic model

1. Droplets are injected into a gas.



2. Euler equations are used for the motion of the gas.
3. A kinetic model is set for droplets.

Unknowns

Introduction

A new model
Context
Hydrodynamic limit
Hydrodynamic limit

Test case
Colliding jets
Crossing jets

Conclusions

- u_p : droplet velocity,
- e_p, T_p : internal energy and temperature of a droplet,
- r : droplet radius,
- ρ_p : droplet density,
- m_p : droplet mass
- $f(t, x, u_p, e_p, r)$: droplet density function ,
- Q : colllision kernel,
- Q_b : break-up kernel,
- ρ : gas density,
- u : gas velocity,
- e : gas internal energy,
- p : gas pressure,
- T : gas temperature,
- α : gas volume fraction,

Constants

- λ : gas thermal conductivity,
- η : gas dynamic viscosity,
- C : gas specific heat,
- $Pr = \dfrac{\eta\, C}{\lambda}$: Prandtl number ,
- Ma :Mach number of the gas,
- $Re = \frac{2r\rho|u_p - u|}{\eta}$: droplet Reynolds number,
- $D_p = \frac{1}{2}\rho\pi r^2 C_d |u_p - u|$: drag coefficient $C_d(\alpha, Re, Ma)$,
- $Nu(Re, Pr)$: Nusselt number, $(Nu = 1 + 0.3Re^{1/2}Pr^{1/3})$
- $\Gamma_p = \dfrac{\partial u_p}{\partial t}$: droplet acceleration ,
- $\phi_p = \dfrac{\partial e_p}{\partial t}$: droplet thermal flux.

Set of equations

$$\partial_t(\alpha\rho) + \nabla_x \cdot (\alpha\rho u) = 0,$$

$$\partial_t(\alpha\rho u) + \nabla_x \cdot (\alpha\rho u \otimes u) + \nabla_x p = -\int_{u_p, e_p, r} m_p \Gamma_p f du_p de_p dr,$$

$$\partial_t(\alpha\rho e) + \nabla_x \cdot (\alpha\rho e u) + p[\partial_t \alpha + \nabla_x \cdot (\alpha u)]$$

$$= \int_{u_p, e_p, r} \left[(m_p \Gamma_p + \frac{m_p}{\rho_p} \nabla_x p) \cdot (u - u_p) - 4\pi r \lambda Nu(T - T_p) \right] f du_p de_p dr,$$

$$\alpha = 1 - \int_{u_p, e_p, r} \frac{4}{3}\pi r^3 f du_p de_p dr = 1 - \int_{u_p, e_p, r} \frac{m_p}{\rho_p} f du_p de_p dr,$$

$$\partial_t f + u_p \cdot \nabla_x f + \nabla_{u_p} \cdot (f \Gamma_p) + \partial_{e_p}(f \phi_p) = Q(f, f) + Q_b(f),$$

$$m_p \Gamma_p = -\frac{4}{3}\pi r^3 \nabla_x p - D_p(u_p - u),$$

$$m_p \phi_p = 4\pi r \lambda Nu(T - T_p),$$

$$p = P(\rho, e),$$

$$e_p = E(T_p).$$

Why a new model ?

- Difficulties arise with small gas volume fractions.
- Lagrangian/Eulerian simulations are costly and badly scalable on a parallelism level : we need to make computations on droplets as less as possible and choose properly the ones we keep.
- Methods of moments for the droplet density functions have proven to be efficient for some applications (work of M.Massot and al. :'Eulerian quadrature based moment models for dilute polydisperse evaporating sprays').

Some remarks on the current model

- For small radii, droplets are rapidly at equilibrium with the gas (velocity/pressure/temperature) due to exchange terms : caracteristic times for droplets are proportionnal to r^2 .

- Droplets can break up so obtaining small radii is frequent : there is always a caracteristic radius (depending on the flow) for which equilibrium can be taken into account.

Basis of the new model

- We separate the phase space of the droplet density function in two parts :
  - the first part is made of droplets at equilibrium with the gas (velocity,pressure and temperature),
  - the second one contains the remaining droplets.
- Droplets at equilibrium are solved through a mix model.
- Others are still solved through a kinetic equation.

$$\partial_t(\rho) + \nabla_x \cdot (\rho u) = 0,$$
$$\partial_t(\rho u) + \nabla_x \cdot (\rho u \otimes u) + \nabla_x p = - \int_{u_p, e_p, r} m_p \Gamma_p f du_p de_p dr,$$
$$\partial_t f + u_p \cdot \nabla_x f + \nabla_{u_p} \cdot (f \Gamma_p) = Q_b(f),$$
$$m_p \Gamma_p = -D_p(u_p - u),$$
$$p = P(\rho).$$

We split f in two parts :

$$f = f_1(t, x, v) + f_2(t, x, v). \tag{1}$$

f_1 (resp. f_2) is a droplet density function for droplets of radius r_1 (resp. r_2).

Toy model (2)

$$\partial_t f_1 + u_p \cdot \nabla_x f_1 + \nabla_{u_p} \cdot (f_1 \Gamma_1) = -F f_1,$$

$$\partial_t f_2 + u_p \cdot \nabla_x f_2 + \nabla_{u_p} \cdot (f_2 \Gamma_2) = F f_1 \left(\frac{r_1}{r_2}\right)^3,$$

$$\Gamma_1 = \frac{D}{r_1^2}(u - u_p),$$

$$\Gamma_2 = \frac{D}{r_2^2}(u - u_p).$$

Let's define $\varepsilon = r_2/r_1$. We get

$$\partial_t(\rho^\varepsilon) + \nabla_x \cdot (\rho^\varepsilon u^\varepsilon) = 0,$$

$$\partial_t(\rho^\varepsilon u^\varepsilon) + \nabla_x \cdot (\rho^\varepsilon u^\varepsilon \otimes u^\varepsilon) + \nabla_x p^\varepsilon$$

$$= \rho_p \int_{u_p} \left(\frac{D \frac{4}{3}\pi r_1^3}{r_1^2} f_1 + \frac{D \frac{4}{3}\pi \varepsilon^3 r_1^3}{\varepsilon^2 r_1^2} f_2 \right) (u_p - u^\varepsilon) du_p,$$

$$\partial_t f_1 + u_p \cdot \nabla_x f_1 + \nabla_{u_p} \cdot (f_1 D \frac{u^\varepsilon - u_p}{{r_1}^2}) = -F f_1,$$

$$\partial_t f_2 + u_p \cdot \nabla_x f_2 + \nabla_{u_p} \cdot (f_2 D \frac{u^\varepsilon - u_p}{\varepsilon^2 {r_1}^2}) = F f_2 \left(\frac{1}{\varepsilon}\right)^3,$$

$$p = P(\rho^\varepsilon).$$

Kinetic energy : limit when $\varepsilon \to 0$ (1)

$$\partial_t \left((\frac{1}{2}\rho^\varepsilon (u^\varepsilon)^2) + \frac{c}{\gamma - 1} (\rho^\varepsilon)^\gamma \right) + \nabla_x \cdot (\frac{1}{2}\rho^\varepsilon (u^\varepsilon)^2 u^\varepsilon) =$$

$$\rho_p u^\varepsilon \left[\int_{u_p} f_1 \frac{D\frac{4}{3}\pi r_1^3}{r_1^2}(u_p - u^\varepsilon) du_p + \int_{u_p} f_2 \frac{D\frac{4}{3}\pi \varepsilon^3 r_1^3}{\varepsilon^2 r_1^2}(u_p - u^\varepsilon) du_p \right],$$

$$\partial_t \int_{u_p} f_1 \rho_p \frac{u_p^2}{2} du_p + \nabla_x \cdot \left(\int_{u_p} f_1 \rho_p \frac{u_p^2}{2} u_p du_p \right) =$$

$$\int_{u_p} f_1 \frac{D}{r_1^2}(u_p - u^\varepsilon) \cdot du_p - \int_{u_p} F f_1 \rho_p \frac{u_p^2}{2},$$

Kinetic energy : limit when $\varepsilon \to 0$ (2)

$$\partial_t \int_{u_p} f_2 \rho_p \frac{u_p^2}{2} du_p + \nabla_x \cdot \left(\int_{u_p} f_2 \rho_p \frac{u_p^2}{2} u_p du_p \right) = \int_{u_p} f_2 \frac{D}{\varepsilon^2 r_2^2} (u_p - u^\varepsilon) \cdot du_p + \frac{1}{\varepsilon^3} \int_{u_p} F f_1 \rho_p \frac{u_p^2}{2}.$$

$$\partial_t \left((\frac{1}{2}\rho^\varepsilon (u^\varepsilon)^2) + \frac{c}{\gamma - 1} (\rho^\varepsilon)^\gamma \right) + \nabla_x \cdot (\frac{1}{2}\rho^\varepsilon (u^\varepsilon)^2 u^\varepsilon) +$$

$$\partial_t \int_{u_p} f_1 \frac{4}{3}\pi \rho_p r_1^3 \frac{u_p^2}{2} du_p + \nabla_x \cdot \int_{u_p} f_1 \frac{4}{3}\pi \rho_p r_1^3 \frac{u_p^2}{2} u_p du_p +$$

$$\partial_t \int_{u_p} f_2 \frac{4}{3}\pi \rho_p \varepsilon^3 r_1^3 \frac{u_p^2}{2} du_p + \nabla_x \cdot \int_{u_p} f_2 \frac{4}{3}\pi \rho_p \varepsilon^3 r_1^3 \frac{u_p^2}{2} u_p du_p =$$

$$-\rho_p \left[\int_{u_p} f_1 \frac{D \frac{4}{3}\pi r_1^3}{r_1^2} (u_p - u^\varepsilon)^2 du_p + \int_{u_p} f_2 \frac{D \frac{4}{3}\pi \varepsilon^3 r_1^3}{\varepsilon^2 r_1^2} (u_p - u^\varepsilon)^2 du_p \right].$$

By letting tend ε to zero (at a constant mass for small droplets *i.e.* $\int_{u_p} f_2 \frac{4}{3}\pi \rho_p \varepsilon^3 r_1^3 du_p = cte$), we must have $\int_{u_p} f_2 \frac{D \frac{4}{3}\pi r_1^3}{r_1^2} (u_p - u^\varepsilon)^2 du_p = O(\varepsilon^2)$, in order to keep kinetic energy positive, so that f_2 is a Dirac mass in velocity (the gas velocity).

New set of equations

Let's $\tilde{\rho}(t,x)$ be defined by $\int\limits_{u_p} f_2 \frac{4}{3}\rho_p \varepsilon^3 r_1^3 du_p$ a partial droplet density, then we get the following system :

$$\partial_t(\rho) + \nabla_x \cdot (\rho u) = 0,$$

$$\partial_t(\tilde{\rho}) + \nabla_x \cdot (\tilde{\rho} u) = \rho_p \frac{4}{3}\pi r_1^3 \int\limits_{u_p} F f_1 du_p,$$

$$\partial_t((\rho+\tilde{\rho})u) + \nabla_x \cdot ((\rho+\tilde{\rho})u \otimes u) + \nabla_x p = -\rho_p \int\limits_{u_p} \left(\frac{D\frac{4}{3}\pi r_1^3}{r_1^2} f_1 \right) (u - u_p) du_p,$$

$$+ \int\limits_{u_p} \left(F \frac{4}{3}\rho_p \pi r_1^3 f_1 \right) u_p du_p$$

$$\partial_t f_1 + u_p \cdot \nabla_x f_1 + \nabla_{u_p} \cdot (f_1 D \frac{u - u_p}{{r_1}^2}) = -F f_1,$$

$$p = P(\rho).$$

Mix model

Let's define $\rho_m = \rho + \tilde{\rho}$, the gas mass concentration $c^+ = \frac{\rho}{\rho}_m$ and the small droplet concentration $c^- = \frac{\tilde{\rho}}{\rho}_m$. We get :

$$\partial_t(\rho_m c^+) + \nabla_x \cdot (\rho_m c^+ u) = 0,$$

$$\partial_t(\rho_m c^-) + \nabla_x \cdot (\rho_m c^- u) = \rho_p \frac{4}{3}\pi r_1^3 \int_{u_p} F f_1 du_p,$$

$$\partial_t(\rho_m u) + \nabla_x \cdot (\rho_m u \otimes u) + \nabla_x p = -\rho_p \int_{u_p} \left(\frac{D\frac{4}{3}\pi r_1^3}{r_1^2} f_1 (u - u_p) \right) du_p de_p dr$$

$$+ \int_{u_p} \left(F \frac{4}{3} \rho_p \pi r_1^3 f_1 \right) u_p du_p$$

$$\partial_t f_1 + u_p \cdot \nabla_x f_1 + \nabla_{u_p} \cdot (f_1 D \frac{u - u_p}{{r_1}^2}) = -F f_1,$$

$$c^+ + c^- = 1,$$

$$p = P(\rho).$$

Hydrodynamic limit for thick sprays

Let's define :

$$\beta = \int_{u_p, e_p, r} \frac{4}{3}\pi r_2^3 f_2 du_p de_p dr,$$

as the volume fraction occupied by small droplets at equilibrium with the gas.

$$\partial_t(\alpha\rho) + \nabla_x \cdot (\alpha\rho u) = 0,$$

$$\partial_t(\beta\rho_p) + \nabla_x \cdot (\beta\rho_p u) = \int_{u_p, e_p, r} F f_1 \rho_p \frac{4}{3}\pi r_1^3 du_p de_p dr,$$

$$\partial_t((\alpha\rho + \beta\rho_p)u) + \nabla_x \cdot ((\alpha\rho + \beta\rho_p)u \otimes u) + \alpha_m \nabla_x p =$$
$$\int_{u_p, e_p, r} m_1 \frac{D}{r_1^2}(u_p - u) f_1 du_p de_p dr + \int_{u_p, e_p, r} F f_1 \rho_p \frac{4}{3}\pi r_1^3 u_p du_p de_p dr,$$

Hydrodynamic limit for thick sprays

$$\partial_t(\alpha\rho e + \beta\rho_p E_p) + \nabla_x \cdot ((\alpha\rho e + \beta\rho_p E_p)u) + p[\partial_t \alpha_m + \nabla_x \cdot (\alpha_m u)]$$

$$= \int_{u_p,e_p,r} \left[(m_p \Gamma_p + \frac{m_p}{\rho_p}\nabla_x p)\cdot(u - u_p) - 4\pi r \lambda Nu(T - T_p)\right] f_1 du_p de_p dr$$

$$+\rho_p \frac{4}{3}\pi r_1^3 \int_{u_p,e_p,r} F f_1 e_p du_p de_p dr + p\frac{4}{3}\pi r_1^3 \int_{u_p,e_p,r} F f_1 du_p de_p dr,$$

$$\alpha_m = 1 - \int_{u_p,e_p,r} \frac{4}{3}\pi r^3 f_1 du_p de_p dr = 1 - \int_{u_p,e_p,r} \frac{m_p}{\rho_p} f_1 du_p de_p dr,$$

$$\partial_t f_1 + u_p \cdot \nabla_x f_1 + \nabla_{u_p} \cdot (f_1 \Gamma_1) + \partial_{e_p}(f_1 \phi_1) = -F f_1 + Q_1(f_1, f_1),$$

$$\Gamma_1 = -\frac{D}{r_1^2}(u_p - u) - \frac{m_1}{\rho_p}\nabla_x p,$$

$$m_1 \phi_1 = 4\pi r_1 \lambda Nu(r_1)(T - T_p),$$

$$p = P(\rho, T),$$

$$E_p = E(T),$$

$$e_p = E(T_p).$$

$$\partial_t(\alpha_m\rho_m c^+) + \nabla_x\cdot\big(\alpha_m(\rho_m c^+ u)\big) = 0,$$

$$\partial_t(\alpha_m\rho_m c^-) + \nabla_x\cdot\big(\alpha_m\rho_m c^- u\big) = \rho_p\frac{4}{3}\pi r_1^3\int_{u_p} Ff_1(u_p-u)du_p de_p dr,$$

$$\partial_t(\alpha_m\rho_m u) + \nabla_x\cdot(\alpha_m\rho_m u\otimes u) + \alpha_m\nabla_x p =$$

$$\int_{u_p,e_p,r} m_1\frac{D}{r_1^2}(u_p-u)f du_p de_p dr + \int_{u_p,e_p,r} Ff_1\rho_p\frac{4}{3}\pi r_1^3 u_p du_p de_p dr,$$

$$\partial_t\alpha_m\rho_m e_m) + \nabla_x\cdot(\alpha_m\rho_m e_m)u) + p[\partial_t\alpha_m + \nabla_x\cdot(\alpha_m u)]$$

$$= \int_{u_p,e_p,r}\left[(m_p\Gamma_p + \frac{m_p}{\rho_p}\nabla_x p)\cdot(u-u_p) - 4\pi r\lambda Nu(T-T_p)\right] f_1 du_p de_p dr$$

$$+ \int_{u_p,e_p,r} F\rho_p\frac{4}{3}\pi r_1^3 f_1 e_p du_p de_p dr + p\int_{u_p,e_p,r} F\frac{4}{3}\pi r_1^3 f_1 du_p de_p dr,$$

Mix model

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$$\alpha_m = 1 - \int_{u_p, e_p, r} \frac{4}{3}\pi r^3 f_1 du_p de_p dr = 1 - \int_{u_p, e_p, r} \frac{m_p}{\rho_p} f_1 du_p de_p dr,$$

$$\partial_t f_1 + u_p \cdot \nabla_x f_1 + \nabla_{u_p} \cdot (f_1 \Gamma_1) + \partial_{e_p}(f_1 \phi_1) = -F f_1 + Q_1(f_1, f_1),$$

$$\Gamma_1 = -\frac{D}{r_1^2}(u_p - u) - \frac{m_1}{\rho_p}\nabla_x p,$$

$$m_1 \phi_1 = 4\pi r_1 \lambda Nu(r_1)(T - T_p),$$

$$p = P(\rho, T),$$

$$E_p = E(T),$$

$$e_p = E(T_p).$$

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- The model is obtained for two sizes of droplet,
- Generally a transition radius r_t is defined because the distribution in droplet sizes is continuous : droplets of radii larger than r_t are dealt with the Boltzmann equation whereas others are put in the mix,
- Other criteria can be added : for instance in combustion at a given temperature droplets vaporize and can be put in the mix.

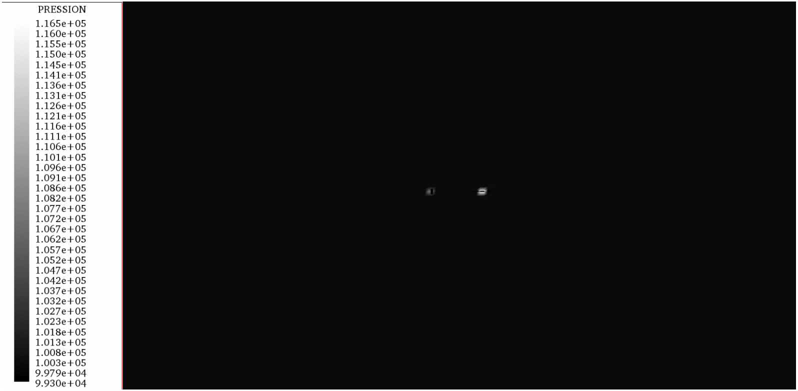
First Test

We present a case of interactions between two jets of metallic droplets moving in the air with the following properties :

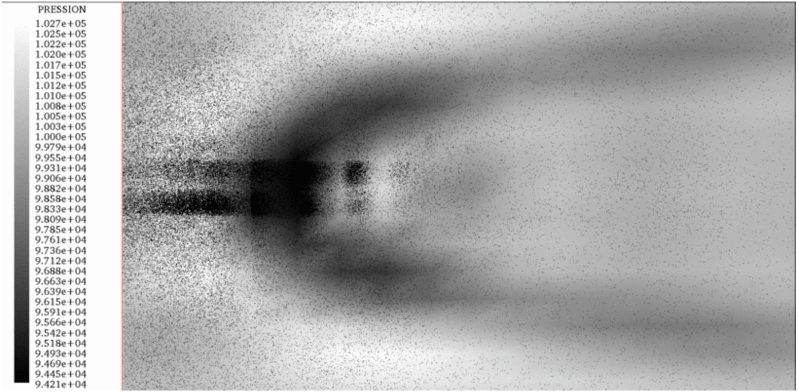
- the initial droplet radius is $6\mu m$,
- the jet coming from the right is two times heavier than the one from the left,
- the jet coming from the right comes faster $-2500 m.s^{-1}$) than the one from the left ($1800 m.s^{-1}$),
- the gas is at rest,
- the gas volume fraction is set between 0.95 et 1 in each jet.

We test different transition radii for droplets and compare results

First Test : results(1)

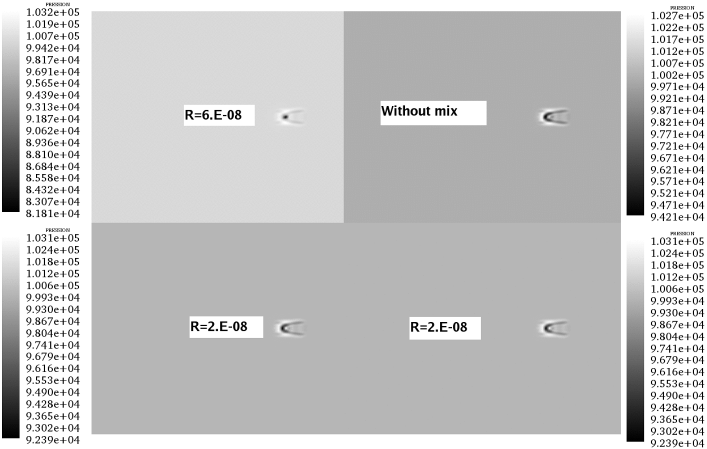


First Test : results (2)

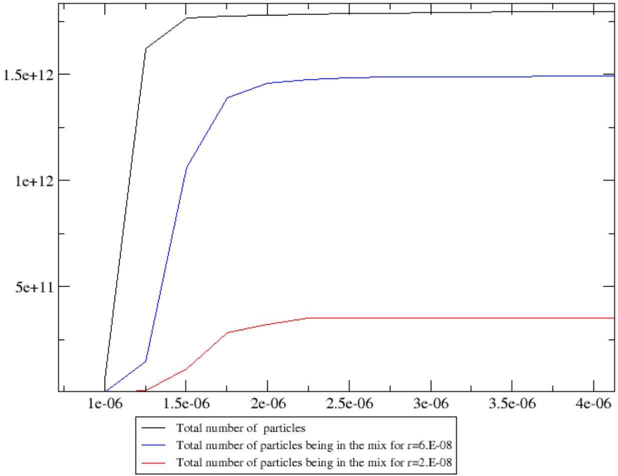


First Test : results (3)

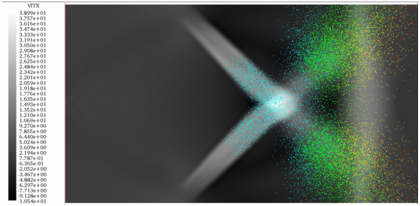
Pressure for different radii of transition r_t
$(r_t = 0\,(\text{without mix}), 20nm, 80nm)$

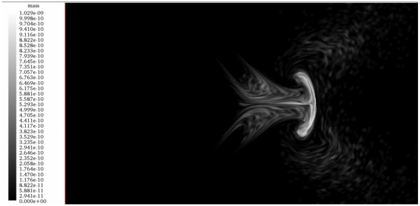


First Test : results (4)

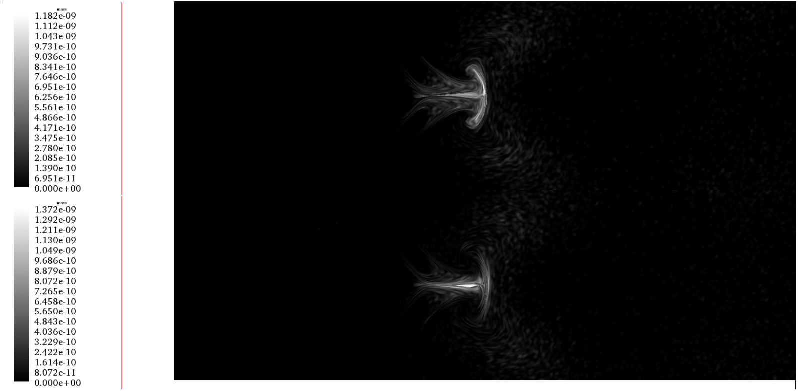


Second Test : crossing jets





Location of droplets mass for the mix model on the top (strict criterium for radius of transition) and droplet model on the bottom



Second Test : results(2)

Location of droplets mass for the mix model on the top (large criterium for radius of transition) and droplet model on the bottom

Conclusions and Perspectives

- The model can deal with various applications.
- More computations are needed to understand the transition towards mix.
- More realistic data should be used for tests.